Prework for Simplified Charged Device Model ESD Tester Modeling tutorial

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This document **reviews** the EE background of the new ESDA tutorial on *Simplified Modeling of CDM ESD Testing* and presents a **simple, practical example** to illustrate the concepts. The reader, and prospective tutorial student, is invited to survey the background material and work through the example. It should connect with something in your academic background and/or work experience, in which case you are well prepared to follow the 90-minute tutorial. Luckily, the concepts involved (relating to linear, time-independent or LTI, systems) are not unique to electrical engineering (EE) majors; they are studied by physicists, mathematicians, statisticians, mechanical and civil engineers (relating to vibrations), and I'm sure I'm leaving out others. Chemistry or Chem E? Probably. I was a physics major in my sophomore year when I first acquired the background reviewed in this document and was, in a word, smitten. It seemed to me that linear circuit models were a good start at grasping and quantifying many physical phenomena, not just electrical.

Now that all of us have easy access to the Internet, my best approach should be to provide **guidance** to reading some worthwhile web-based articles, in this case mostly on Wikipedia. Otherwise, I'm just cutting, pasting and editing material that is "out there" and available to all.

Complex Impedances

Central to the ultimate simplified model of the CDM tester is the notion of linear (i.e., R, L and C) circuit models in the complex frequency ($s=\sigma+j\omega$) domain. This first article (we'll call it #1) takes a while to get there, but it does arrive at complex frequencies in the section called "Generalised s-plane impedance" (hmm, no "z" in the first word; must be from the UK): http://en.wikipedia.org/wiki/Electrical_impedance. And of course the outcome is

Element	**Impedance expression**
Resistor	R
Inductor	sL
Capacitor	1/sC

Laplace Transforms

The critical sentence leading up to the impedance table is "Signals are expressed in terms of complex frequency by taking the Laplace transform of the time domain expression of the signal." Oh, so that suggests that we can also consider the Laplace transform of the signal itself, without necessary reference to linear circuit elements, and transform it into the complex frequency domain, i.e., acquire a spectrum. It also suggests that time-dependent voltage and current sources have their Laplace-transformed equivalents, so it's looking like we can **build an entire circuit** and cross back and forth between time and frequency domains at will, using these tools. Sounds like it's worth reading up on the Laplace transform, in article #2: http://en.wikipedia.org/wiki/Laplace_transform. Whew! Time to focus on the essentials and not get hung up on mathematical rigor or other applications—regions of convergence, contour integrals, subtle differences between Laplace and Fourier transforms (aside from 90° rotation), statistical applications, and all that are important but we don't have to sweat it now. We should focus on a few simple implications of the basic definition of the Laplace transform,

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt. \quad (1)$$

First of all, as stated in article #1 and above, the complex impedances of R, L and C are derived from their I-V relations. The tables in article #2 carry this out for a number of functions that could serve as signal sources, sinusoidal and exponential of course, but also the unit step function u(t) (sometimes written as $u_{-1}(t)$), which transforms to 1/s. You see, that's why the spectrum of a step function goes 1/f!

Differential and Integral Operators, Convolution, Filtering

Differentiate a step function and that gives an infinite impulse of 0 length at t=0, also called δ(t); what is its transform? The table says 1, and it's pretty obvious from integrating (1) over a delta function, but the tables also associate "s" with a derivative, i.e., you multiply 1/s by s and you should have the transform of δ(t). Indeed you do; the spectrum of δ(t) is famously flat over all frequencies. While we're at it, note that the integral (inverse of derivative) transforms to 1/s, meaning that integrating a function means multiplying its transform by 1/s. So the integral and differential operators have their s-domain equivalents. [That's just the beginning of a discussion of operational calculus, if interested see this article, #3, and don't be upset if s turns into p: http://en.wikipedia.org/wiki/Operational_calculus.] Note in the tables of #2 that integration also means convolving the step u(t) with the time domain function f(t). This is a powerful theorem, showing that multiplying in the s-domain means convolution in the time domain, per this expression:

$$(f * g)(t) = \int_0^t f(\tau) g(t - \tau)\, d\tau \Leftrightarrow F(s) \cdot G(s) \quad (2)$$

So now you can, for example, describe a **filter** in the frequency domain and find its time domain response to a signal of your choice, starting with steps and impulses. Some background, and some lucid ways to visualize convolution, is covered in articles #4 and #5, as follows:

http://en.wikipedia.org/wiki/Convolution
http://reference.wolfram.com/legacy/applications/signals/MathematicalUtilities.html

My favorite way to visualize convolution is the **"flip and slide"** mnemonic, and the Wolfram article #5 actually calls it that and shows it in a sequence of pictures of asymmetric functions being convolved, almost like animation. The Wikipedia article #4 has actual animation but the "flipped" function is symmetric. They also show two RC decay functions being convolved but without animation, just flipped, and slid, and you can see the overlap and imagine the integral but the final function is not plotted! Too bad, because our basic HBM waveform is also two convolved RC decays, i.e., the current function has two real, negative poles, giving us our famous "double exponential" HBM waveform with 2-10 nsec rise time and 150 nsec decay time. So it's not hard to imagine the final result of the Wiki article's RC decay convolution example.

Practical Example:

With that background, let's work through a simple RLC example from "real life", back in the 1980s when we were building one of our first manually-operated transmission line pulse (TLP) systems at Intel. The front end of the TLP charged cable voltage source can be seen as a step voltage with 50 ohm source impedance. Our target, usually a low resistance protection device, typically has impedance far below 50 ohms, so any voltage measurement method across the device will be looking at a voltage source with very low output impedance; let's say it's about 2 ohms. This is fine for a probe, of course, where you want probe Z high enough so that the signal is not disturbed. So Zprobe can be "high" enough for many TLP targets of interest if Zprobe is just a few hundred ohms.

Our first probe in the 1980s TLP system, however, was a Tektronix 6009 100:1 scope probe with 10 Megohm input impedance, as it was good to high voltage and offered such attachments as a BNC adaptor, for easy plug-in to a signal source, where high impedance and no disturbance of the signal source was desired. A BNC tee, for example, could be used with the Tek 6009 so equipped, to sample a signal along a 50-ohm cable, and there would be minimal capacitive loading. I believe the catalog gave the input capacitance of the 6009 probe with BNC fixture as 2 pF; probe bandwidth was generally known to be around 125 MHz, maybe a little higher because I might also be remembering the influence of some of our slower scope plug-ins of that era! The probe was not much better than 125 MHz, for sure.

One day I came into the lab and found that the lab tech had built a box with a BNC panel connector, a F-BNC connector suitable for plugging in the M-BNC-equipped Tek 6009 probe. But the ground and center conductor of the BNC panel mount had to be wired to the TLP device and setup, in order to "tap" the voltage across the ($Rd \approx 2$ ohms) TLP target. Several inches of thick wire were used to get to and from the BNC panel mount to the TLP device connections, as shown in Figure 1.

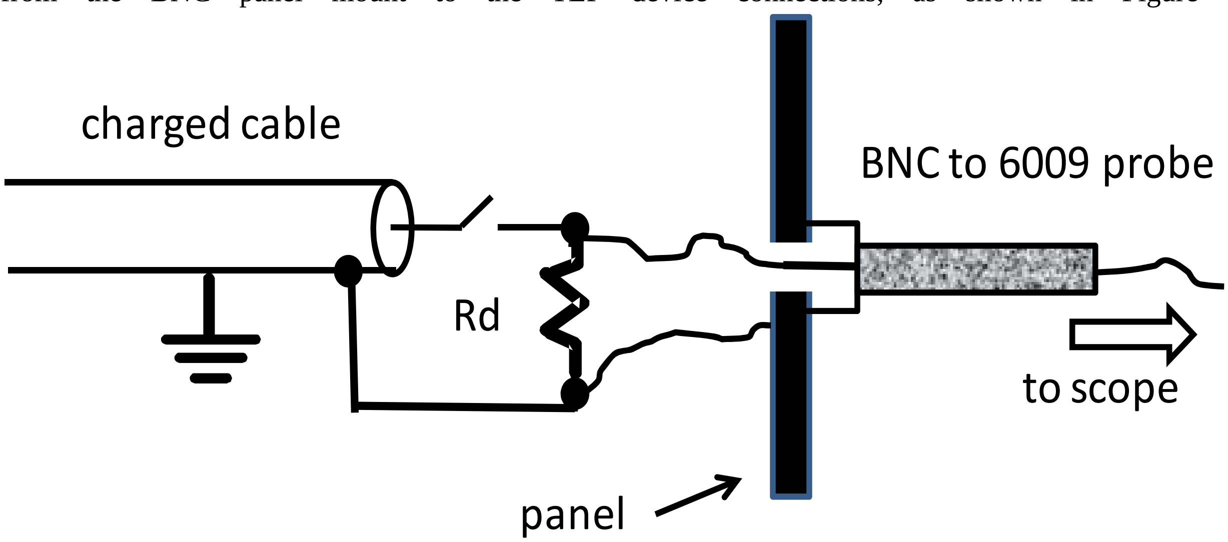

Figure 1. Setup with 10 Megohm 100:1 voltage probe on panel mount with wires to TLP target.

I was a little concerned about the wire length, but this was a lot easier to set up and use than the 6009 with clumsy probe clips. However, when we tried out a TLP waveform on a device or low-ohm resistor, we got vicious ringing at the initial step, ringing that took many nanoseconds to settle out (see Figure 2). So it was very hard to read the TLP response. What was the problem; was this real? It didn't help to shorten the device or resistor wires.

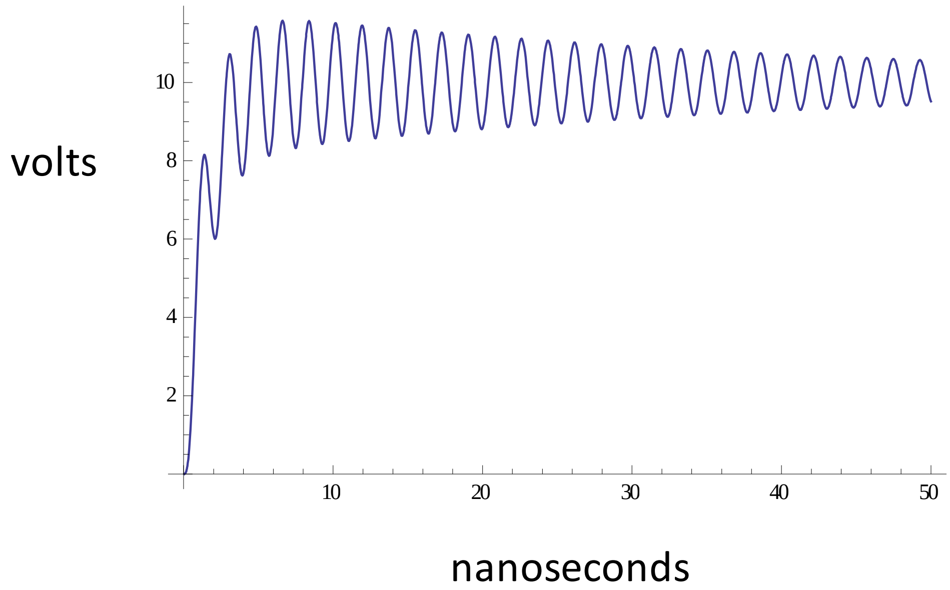

Figure 2. This is what the scope trace looked like at the front end of a TLP pulse with a low resistance target as in Fig. 1. Why the ringing?

Pretty soon it became clear that the primary TLP circuit was not misbehaving at all. What we faced at the front end of the TLP pulse was a circuit as pictured in Figure 3.

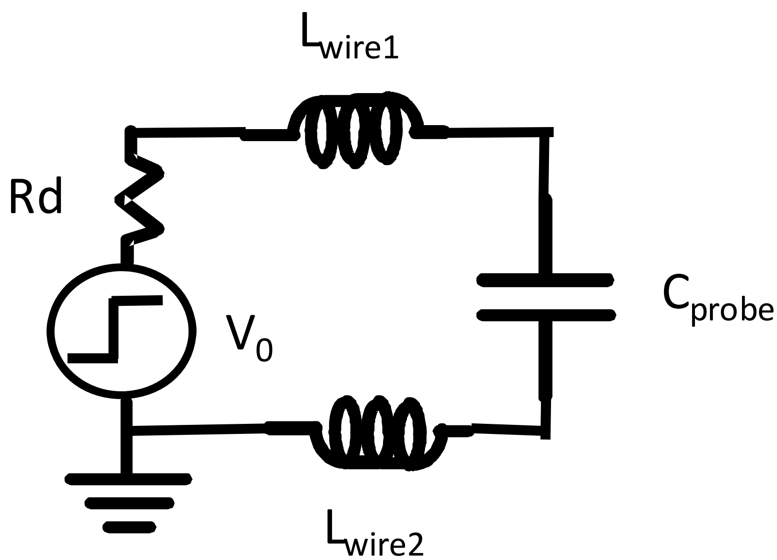

Figure 3. Equivalent circuit of the probe loop from Fig. 1. 10 Megohm probe resistance across C_{probe} has negligible effect; the probe effectively measures voltage across C_{probe}.

Because the L_{wire} elements come from several inches of wire, we can apply the usual ham radio operator's rule of thumb of 10-20 nH per inch to estimate total $L_{wire}=L_{wire1}+L_{wire2}$, so let's set L_{wire} at 40 nH, to go with the 2 pF of C_{probe}. With device or resistor Rd of only a few ohms, it's becoming clear that the probe circuit has high Q and is ringing independent of the TLP loop. We will soon calculate and prove that. Before we proceed further, what would you do to restore the measured voltage across C_{probe} to something approaching the true voltage across Rd? We'd like to preserve the convenience of this probe arrangement. Read the next paragraph to continue.

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You are correct if you suggest that an extra series resistor in the wire loop will damp the oscillations and also produce a measured voltage at C_{probe} that resembles the true voltage across Rd. Suppose we want to adjust the Fig. 3 circuit accordingly and prove that the result is acceptable. Remember, the probe starts to roll off at 125 MHz, so reducing the extra probe circuit to a minor influence should be possible.

What should the total loop resistance be in order to damp the probe circuit "enough"? Let's decide to reduce the damping to the slightly underdamped case (Fig. 2 is extremely underdamped), with a damping factor D=0.707 or $1/\sqrt{2}$. This is a good choice because it is the "pseudo-Gaussian" 2-pole response, as described in an important paper from 1999 [1]. Also, the two poles in the solution have real and imaginary parts whose magnitudes are equal; in other words, for natural frequencies $s\pm = \sigma \pm j\omega$, $|\sigma|=\omega$.

So now the problem for you the reader to solve is, what total resistance Rd+Rs would tame that probe loop and make the probe voltage across C_{probe} (2 pF) more readable and more representative of the TLP-generated voltage across Rd, given total L_{wire}=40 nH? For Rd=2 ohms, let's suppose that we charge the 50-ohm line to 260V, meaning that we expect the final voltage Vo to be 10V. So the step function (we'll overlook TLP rise time for now, although I used about 2 nsec to produce Fig. 2) is Vo/s. The voltage divider equation will have a quadratic in the denominator, for which the complex conjugate pair of roots will satisfy $s\pm = \sigma \pm j\omega$, $|\sigma|=\omega$, as above.

See next page for answer. DON'T LOOK until you calculate something. If you know your damping factors, it's extremely easy. Otherwise, pull the right things from the quadratic solution and you're done.

Answer:

Using complex impedances, the voltage divider relation gives the following expression for the probe voltage if we let L_{wire}=L and C_{probe}=C:

$$V_{Cprobe}(s) = \frac{V_0}{s}\frac{1/sC}{Rd + Rs + sL + 1/sC} = \frac{V_0}{s[1 + (Rd + Rs)Cs + LCs^2]} \quad (3)$$

This is fundamentally a step function Vo/s, modulated by a 2-pole filter with poles corresponding to the quadratic solutions. The final voltage will of course be Vo, 10V as discussed above for the case of Rd=2 ohms. If R=Rd+Rs, we can find the solutions such that s± = σ±jω, |σ|=ω for D=0.707 by setting "-b" in the equation (RC) to the discriminant ($\pm\sqrt{(R^2C^2-4LC)}$). Square both sides and R can be found from

$$R^2C^2 = 4LC - R^2C^2, \text{ or } R^2 = 2\frac{L}{C}. \quad (4)$$

Solving for the case of L=40 nH and C=2 pF, R=200 ohms. So we should add 198 ohms to our Rd. In practice, there's a vast improvement even from adding 100 ohms. See Figure 4 for configuration.

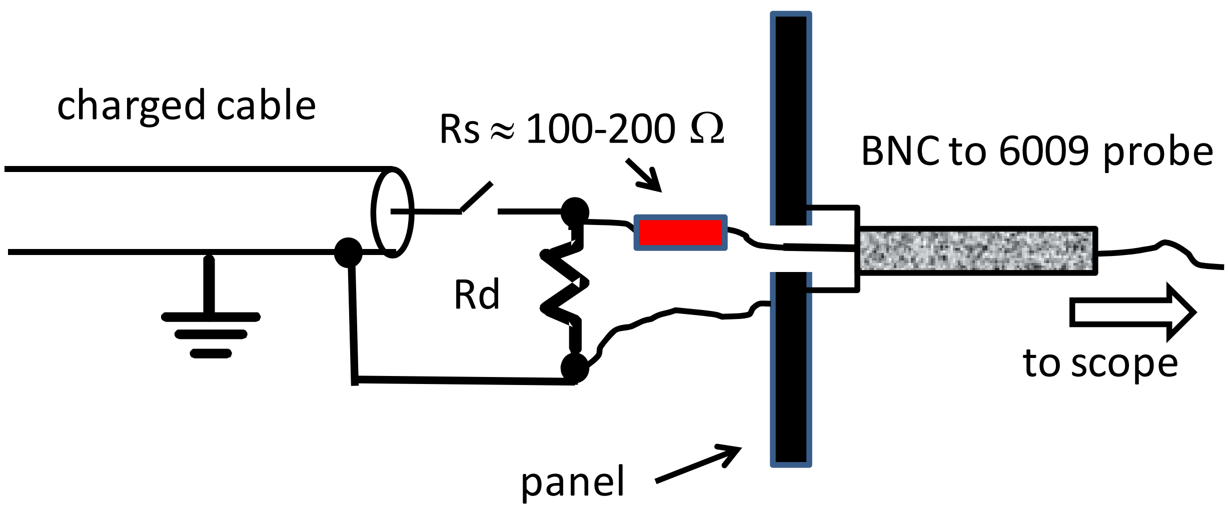

Figure 4. Damping resistor Rs added to current probe measurement loop.

The "easy" way to solve this problem, if you know your RLC damping factors, is to recall that damping factor D is such that $D = \frac{RC}{2\sqrt{LC}}$. Knowing our target D=1/√2 and the values of L and C give R=200 ohms readily.

Simulated waveform results for total resistance of 200 ohms and 100 ohms are shown in Figure 5.

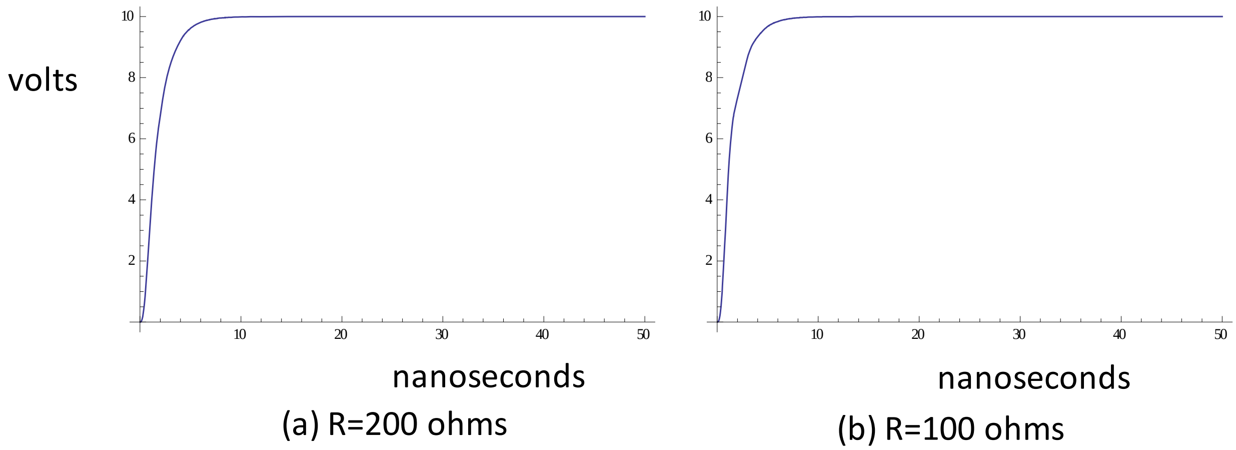

Figure 5. Probe waveforms, simulated at the same time scale as Fig. 2 with listed values of total resistance. TLP 10-90% rise time is about 2 nanoseconds.

As you can see, the waveform for 100 ohm resistance is almost identical to the one for our calculated value of 200 ohms. In our "real life" lab, 100 ohms worked extremely well.

It can be shown that the impact of the probe circuit on the total measurement is not severe. Even with R=200 ohms, the RC product (Elmore delay) of the probe circuit is 400 picoseconds. But we said the probe rolls off at 125 MHz. If that is the 3 dB point and the probe response is the same kind of pseudo-Gaussian (D=0.707) we discussed above, then [1] tells us that the 10-90% rise time of the intrinsic probe is 2.717 nanoseconds. That adds in quadrature with the 10-90% rise time of the probe circuit we just designed (604 psec, scaled up from RC according to formulae in [1]) to give a total probe circuit rise time of 2.783 nsec. So the extra probe circuit has only a 2.4% bandwidth effect for 200 ohms, 1.2% with the 100 ohm resistance.

There are more discussions like the above, following from Reference [1], in the more recent Reference [2].

References:

[1] C. Mittermayer and A. Steininger, "On the Determination of Dynamic Errors for Rise Time Measurement with an Oscilloscope", IEEE Trans. on Instrumentation and Measurement, Vol. 48, no. 6, pp. 1103-07, Dec. 1999.

[2] T.J. Maloney and A. Daniel, "Filter Models of CDM Measurement Channels and TLP Device Transients", 2011 EOS/ESD Symposium Proceedings, pp. 386-394. See https://sites.google.com/site/esdpubs/documents/esd11.pdf

See also web articles #1-5, cited in text.

See you in Seattle!