Total Charge Theorem for Directional Couplers and Z-Matched Coupled Lines

Timothy J. Maloney, *Senior Member, IEEE,* and Steven S. Poon

***Abstract*—It is proven that, in a two-line directional coupler of conventional design, the response to a voltage step on the input is a net amount of coupled charge on the output that is constant, and depends only on fixed properties of the coupler and the size of the voltage step, not on the waveform. This general property of Z-matched coupled lines is useful for pulse creation, and provides insight into certain memory bus techniques that use directional couplers for impedance-matched data transmission.**

***Index Terms*—Coupled transmission lines, crosstalk, directional couplers, pulse generation, transmission line theory.**

I. Introduction

THE two-line quarter-wave directional coupler, ideal in theory and near-ideal in manufacture, operates over about an octave of bandwidth and has for many decades been considered in the frequency domain. Most of the basic theoretical and experimental work was completed by the 1960s [1]; Cohn and Levy summarized the history of directional couplers in an excellent 1984 review article [2]. But today's digital age has led to new applications for directional couplers, to be discussed later, and there is interest in such topics as the transient response of directional couplers. There is of course a parallel literature on crosstalk, also going back many decades [3], [4], which deals with the very same coupled line equations in the time domain, but it also covers aspects of undesired coupling such as weak coupling approximations, multiple lines, dielectric inhomogeneity, and impedance mismatches [3]–[7]. The present work considers some aspects of the step response of a well-engineered directional coupler, with coupling of any strength.

It is well known that the coupled line equations [6], [7] for two lines are solved by two eigenmodes, an odd mode and an even mode [1]–[7]. The ideal directional coupler has even and odd modes of equal velocity (meaning fields in a homogeneous dielectric, as in stripline), and overall impedance matching to a system impedance of Z_0.

Coupled line theory [1] gives the induced voltage on the coupled line of a directional coupler as

$$\frac{V_2}{V_1} = \frac{jk\sin\theta}{\sqrt{1-k^2}\cos\theta + j\sin\theta}. \quad (1)$$

$k = (Z_{oe} - Z_{oo})/(Z_{oe} + Z_{oo}), \theta = (\omega\pi)/(2\omega_0)$, and ω_0 is the $1/4$ wave frequency. Z_{oo} and Z_{oe} are the odd and even mode impedances, and all four ports are presumed to be impedance

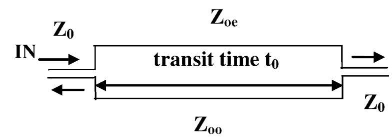

Fig. 1. Directional coupler impedance scheme, with input and coupled ports on the left, thru and isolated ports on the right. When $Z_0 = \sqrt{Z_{oe}Z_{oo}}$, the isolated port is null.

matched, most often to 50 Ω. The condition $Z_0 = \sqrt{Z_{oe}Z_{oo}}$ means that capacitive and inductive coupling are balanced such that they add for the coupled port of the second line, and cancel for the isolated port of the second line.

However, this well-known frequency domain treatment does not offer much insight into the step response of the coupler. While mode analysis has been used to formulate very general expressions for the time-domain response of two coupled lines [6], [7], we aim to use similar tools to derive some concise and useful properties of the well-matched directional coupler.

II. Total Charge Theorem

Consider a two-line directional coupler with system impedance Z_0 and the impedance-matched condition $Z_0 = \sqrt{Z_{oe}Z_{oo}}$, where Z_{oe} and Z_{oo} are the even and odd mode impedances respectively. Under these conditions, the even and odd modes and their reflection coefficients are complementary, so that at all frequencies the input port is impedance matched (no reflection because reflected even and odd modes cancel) and the isolated port is null. Thus, a single reflection coefficient

$$\rho = \frac{Z_{oe} - Z_0}{Z_{oe} + Z_0} = \frac{Z_0 - Z_{oo}}{Z_0 + Z_{oo}}$$

is sufficient to describe the response to an abrupt traveling wave voltage step ΔV at the input port, as that step can be decomposed into equal parts even and odd mode signals, and the mode reflection coefficients with respect to Z_0 are of opposite sign. Note the distinction between this ρ and the coupling factor k, above; here the mode impedances are compared to their geometric mean Z_0 instead of to each other.

In terms of $k, \rho = (k)/(1+\sqrt{1-k^2})$, so for a $k = (\sqrt{2})/(2)$ coupler of 3 db at midband, $\rho = \sqrt{2} - 1$.

Fig. 1 sketches the mode impedance as it splits into complementary even and odd modes over the length of the coupler and then becomes Z_0 again beyond the coupled lines. The reflection and transmission coefficients at the two interfaces determine the response for each time step. The response at the coupled port to an abrupt total voltage step ΔV at the input port is an infinite series of timed steps, starting with $V_1 = \rho\Delta V$ for the first time step and then reduced at each succeeding time step by an amount due to transmission and reflection at the two interfaces with Z_0

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The authors are with Intel Corporation, Santa Clara, CA 95052 USA (e-mail: timothy.j.maloney@intel.com; steven.s.poon@intel.com).

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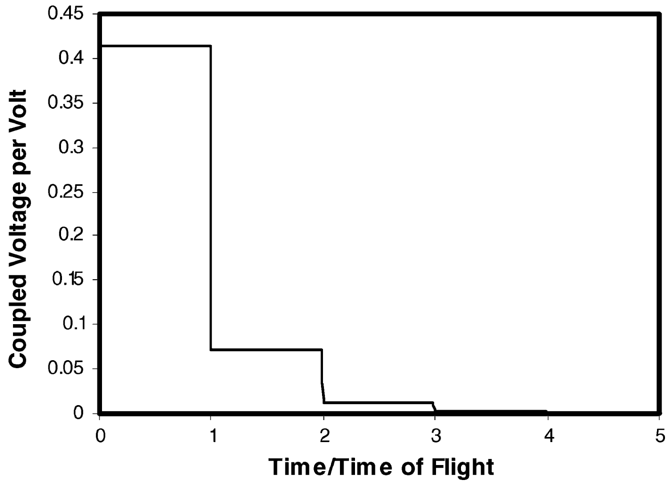

Fig. 2. Ideal step response, on the coupled port, of a 3-db (k =0.707) directional coupler, to an abrupt 1 V traveling wave step on the input port. One time step is $2t_0$.

impedance. It takes one "time step" to sense the abrupt transition back to Z_0 at the far end of the coupler, so a time step is the round trip propagation time for the coupled line section; let this time be $2t_0$.

Understanding that the odd mode is wholly complementary to the even mode and adds an equal amount to the coupled voltage, let us focus on the even mode to solve for the wave series. For the even mode, the reflection coefficients at the left and right interfaces in Fig. 1 are ρ and $-\rho$, respectively, while the transmission coefficients for right-going and left-going waves at the left-hand interface are $\tau_1 = (2Z_{oe})/(Z_0 + Z_{oe}) = 1 + \rho$ and $\tau_2 = (2Z_0)/(Z_0 + Z_{oe}) = 1 - \rho$, respectively.

After one time step the coupled port has an additional term, $-\tau_1\tau_2\rho\Delta V$, due to the first reflection at the right-hand interface, reducing the total voltage. Each succeeding time step has an additional term like the above but with an additional factor of ρ^2, due to two more interfacial reflections. Thus the general expression for the total coupled voltage during the nth time step is

$$V_n = \rho\Delta V \left(1 - \tau_1\tau_2(1 + \rho^2 + \rho^4 + \cdots + \rho^{2n-4})\right)$$
$$= \rho\Delta V(1 - (1-\rho^2)(1 + \rho^2 + \rho^4 + \cdots + \rho^{2n-4})), \quad \text{for } n \geq 2. \quad (2)$$

The truncated series can be captured using standard methods so that

$$V_n = \rho\Delta V \left(1 - \frac{(1-\rho^2)(1-\rho^{2n-2})}{(1-\rho^2)}\right) = \Delta V \rho^{2n-1}, \quad \text{for } n \geq 1. \quad (3)$$

This response to an abrupt step is plotted for a 3 db coupler in Fig. 2. Now that we have the voltage time series, the current can be found for each time step and summed to give total charge

$$Q = \frac{\rho\Delta V(1 + \rho^2 + \rho^4 + \cdots)}{Z_0} 2t_0 \quad (4)$$
$$= \frac{\rho\Delta V}{(1-\rho^2)Z_0} 2t_0.$$

This is also

$$Q = \frac{k\Delta V}{\sqrt{1-k^2}Z_0} t_0. \quad (5)$$

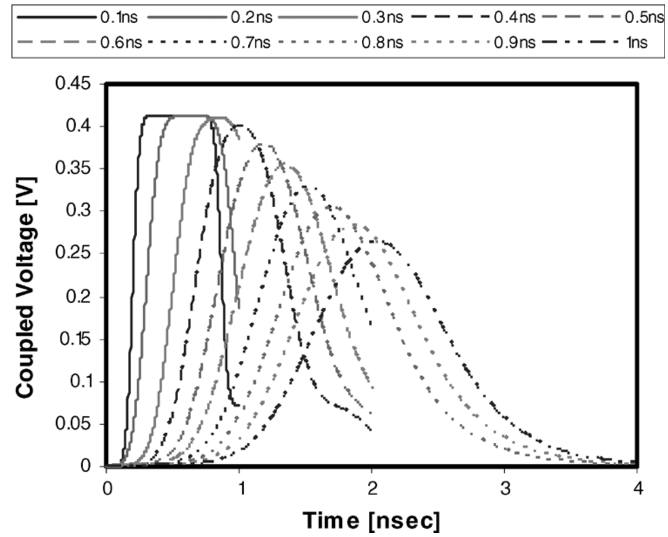

Fig. 3. Coupled voltage waveforms of a 3-db (k =0.707) directional coupler with 2-V error function input sources and different rise times, resulting in 1-V traveling waves on the input. The quarter-wave frequency of the coupler is 750 MHz, time-of-flight ($2t_0$) is 667 psec. Area under the curves, divided by 50 Ω, is always 6.67 pC, or 6.67 pC/V, in agreement with the Total Charge Theorem.

It is clear that (4)–(5) apply to the net charge coupled by a net voltage step ΔV of arbitrary form, i.e., $\Delta V = V(t) - V(0)$ where $V(t)$ is the final voltage. Any signal resulting in a net change ΔV can be broken down into a set of infinitesimally small abrupt steps, each of which has the multiplier of (4)–(5). All of these terms then sum to the same net effect as expressed in (4)–(5)

III. Computer Simulations, Laboratory Experiments, and Applications

Computer simulation of the 3-db coupled-line system for several step waveforms confirmed that the area under the response curve is constant, depending only on the difference between initial and final voltage. The following step voltage source is in series with the line impedance:

$$v_{\text{input}} = \text{erf}\frac{1.812(t - t_c)}{t_r} + 1. \quad (6)$$

In this equation, t_r is the 10%-to-90% rise time of the input voltage source and t_c is the center position. The result is the set of response functions in Fig. 3. The limit of slow rise time is of course the derivative, a Gaussian function. In each case, the area under the curve integrates to 6.67 pC for a 50-Ω system and 750 MHz center frequency, as expected from the Total Charge Theorem for a 1-V traveling wave step.

In the laboratory, several high voltage couplers, 3 db or stronger, with center frequencies of 500–750 MHz, were built and tested for high-voltage impedance-matched pulse applications, and the Total Charge Theorem was confirmed to high accuracy at all voltage levels [8]. This is to be expected because at these frequencies, stripline couplers can be nearly perfect, with over 30 db of directivity. The work in [8] occasioned the discovery of the Total Charge Theorem, when it was found that directional couplers can be used to generate Z-matched pulses resembling the charged device model (CDM) of electrostatic discharge (ESD) [9], [10]. More experimental results, and more computer simulations, appear in recent work [8], [11] addressed primarily at CDM ESD pulse generation. The coupled line method of generating Z-matched pulses is particularly applicable to this subject because CDM ESD is created by excess charge density (related through Gauss' Law to electric field) on a component. When joined with the area of field capture of the component, a given amount of charge thus expresses the strength of a CDM ESD event. The Total Charge Theorem then tells us that the desired charge packet directly relates to a voltage step of a given height through the coupler. The coupler design and the shaping of the step otherwise determine the pulse. A voltage step modified by a rise time filter, for example, will not change the charge quantity, aside from dc insertion loss of the filter.

Another application arises from the use of directional couplers in high-speed digital multidrop memory busses [12], [13]. In these schemes, the Total Charge Theorem gives the size of the charge packet used to transfer signals to multiple drop points with Z-matching. The theorem should also prove useful in considering receiver response and in calculating bit error rates for these systems [14].

IV. Conclusion

We have derived a Total Charge Theorem for directional couplers and impedance-matched coupled pairs of lines that accurately predicts the total charge in the coupled line that results from a voltage step on the input. It is a function only of the voltage step height, the coupling constant and electrical length of the coupler, and the system impedance. Thus a charge packet of fixed size can be expected from a given incoming voltage step, regardless of exact waveform. The Total Charge Theorem is a general property of directional couplers and comes from consideration of the time domain instead of the frequency domain, much like certain crosstalk formulations of coupled lines but with particular attention to step response. The Total Charge Theorem is likely to find use in any application of directional couplers to pulses or digital signals, and provides a useful tool for understanding and analysis.

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