Tapered RC Networks for Thermal Modeling of ESD

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Biography

Timothy J. Maloney received an S.B. degree in physics from the Massachusetts Institute of Technology in 1971, an M.S. in physics from Cornell University in 1973, and a Ph.D. in electrical engineering from Cornell in 1976, where he was a National Science Foundation Fellow. He was a Postdoctoral Associate at Cornell until 1977, when he joined the Central Research Laboratory of Varian Associates, Palo Alto, CA. At Varian until 1984, he worked on III-V semiconductor photocathodes, solar cells and microwave devices, as well as silicon molecular beam epitaxy and MOS process technology. From 1984 he was with Intel Corp., Santa Clara, CA, where he was concerned with integrated circuit ESD protection, CMOS latchup testing, fab process reliability, signal integrity, system ESD testing, and design and testing of standard IC layouts. He was a Senior Principal Engineer at Intel from 1999 until retirement in June of 2016. He received the Intel Achievement Award for his patented ESD protection devices, which have achieved breakthrough ESD performance enhancements for a wide variety of Intel products. He now holds forty patents.

Dr. Maloney received Best Paper/Outstanding Paper Awards for his contributions to the EOS/ESD Symposium in 1986, 1990, and 2015, was General Chairman for the 1992 EOS/ESD Symposium, and received the ESD Association's Outstanding Contributions Award in 1995. He has taught short courses at UCLA, University of Wisconsin, and UC Berkeley. He is co-author of a book, "Basic ESD and I/O Design" (Wiley, 1998), and is a Life Fellow of the IEEE.

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Abstract

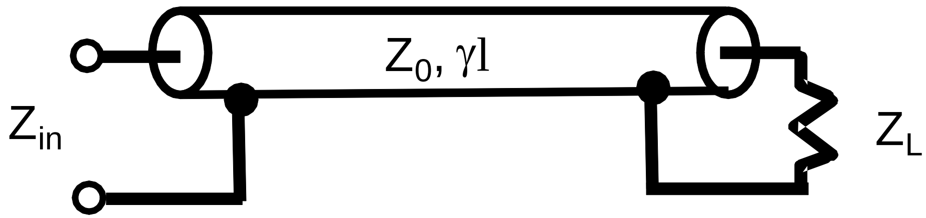
- Distributed R-C line segments with any taper can be modeled simply with a spreadsheet using recursive expressions in the complex frequency domain. This augments the R-C ladder circuit for thermal modeling. The pulse power (Dwyer) curve departs from Wunsch-Bell (exponent $n=0.5$) primarily because of heat spreading, resulting in lumped or distributed RC thermal networks for devices that have smaller R and larger C with depth. We prove that tapered transmission line theory shows that $0<n<0.5$ in the heat spreading case, in agreement with much data. But when $0.5<n<1$, it is likely due to adiabatic heating of a metal layer at the surface by the heat source, as the adiabatic layer behaves like a lumped capacitance and affects the power curve for short and intermediate times. For extremely long times, a deep heat sink or spreading of heat to a large volume will often cause the power curve to converge asymptotically to a constant value.

Objective and significance

- We need to synthesize simple RC thermal network models to fit observed ESD heat pulse data for semiconductor devices.
- Tapered thermal RC networks in paper 3A.3 by C. Russ et al. at EOS/ESD 2018 (Russ18) produce power curve exponents (see below) greater or less than 0.5 depending on whether impedance (Z)-related R/C grows or shrinks with depth from the heat source.
- Here we use similar RC transmission line segments with tapered Z, varying RC and employ a simple recursive spreadsheet calculation to find final thermal Z(s).
- Surface adiabatic effects are modeled with an initial capacitance (Z(s)~1/Cs).
- If Z(s) goes as $1/s^n$, the related Dwyer power curve goes as $1/t^n$.
- The segmented t-line models are shown to be equivalent to cases in Russ18 for n=0.41 and n=0.7.
- Similarly, steady state is reached at s below the (inverse) time constant of the entire structure, as expected.
- The theory of continuously tapered t-lines confirms observations of n and suggests more accurate fitting functions for observed data.

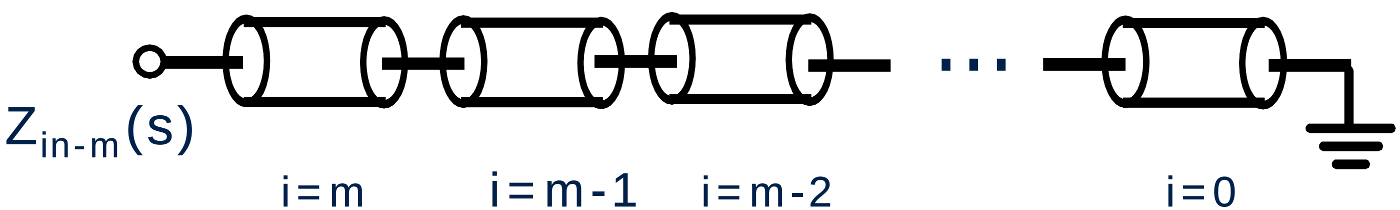
Tapered Chain of R-C T-lines

General formula:



$$Z_{in} = Z_0 \left[\frac{Z_L + Z_0 \tanh(\gamma l)}{Z_0 + Z_L \tanh(\gamma l)} \right]$$

$$Z_{0i} = \sqrt{\frac{R_i}{C_i s}} \qquad \gamma_i = \sqrt{R_i C_i s}$$

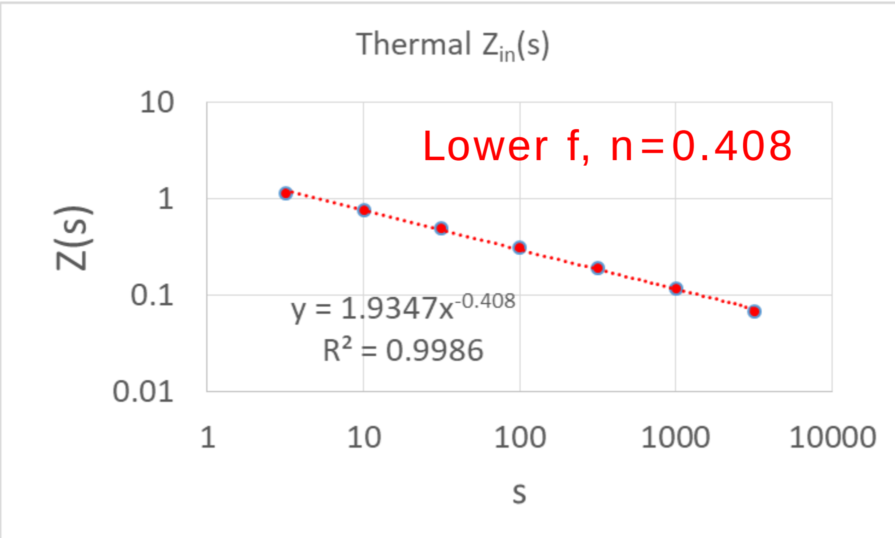


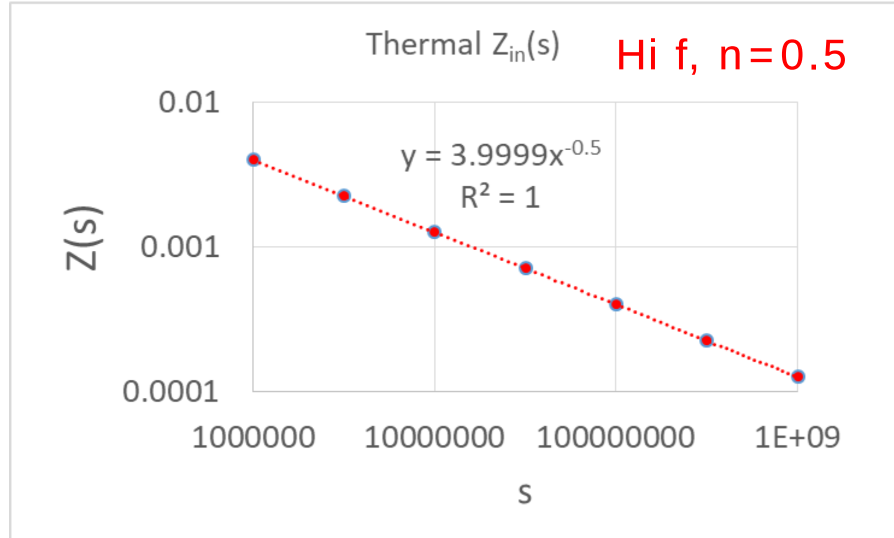
$Z_L = 0$ is perfect heat sink

Taper: $R_{i+1} = rx * R_i$ $C_{i+1} = cx * C_i$

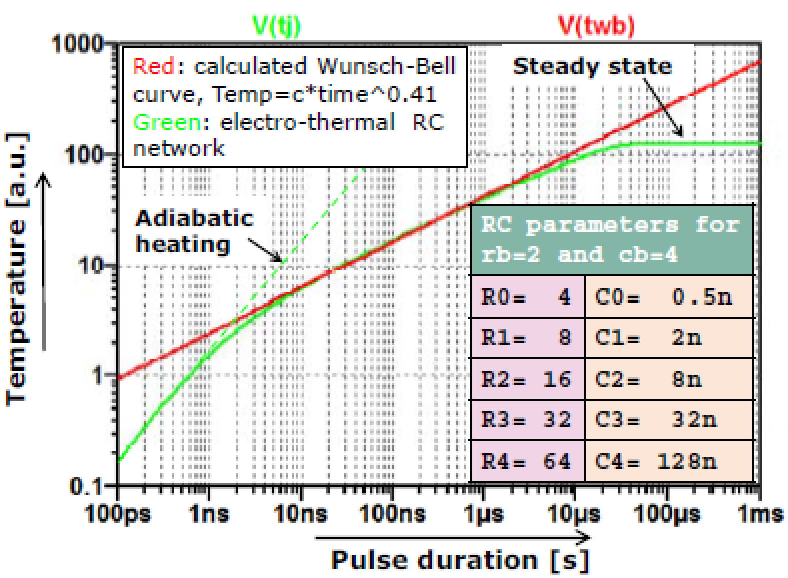
- Input Z(s) recursively calculated for 8 segments (m=7); use normalized $1 = R_0/C_0 = R_0 C_0 = \ell_i$

Matching of power law at mid-range of s





- 8-stage t-line equivalent to R-C series from Russ18 (3A.3, EOS/ESD18), where n=0.41



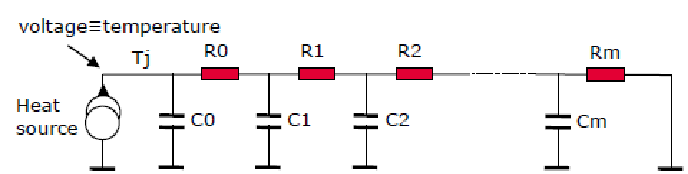


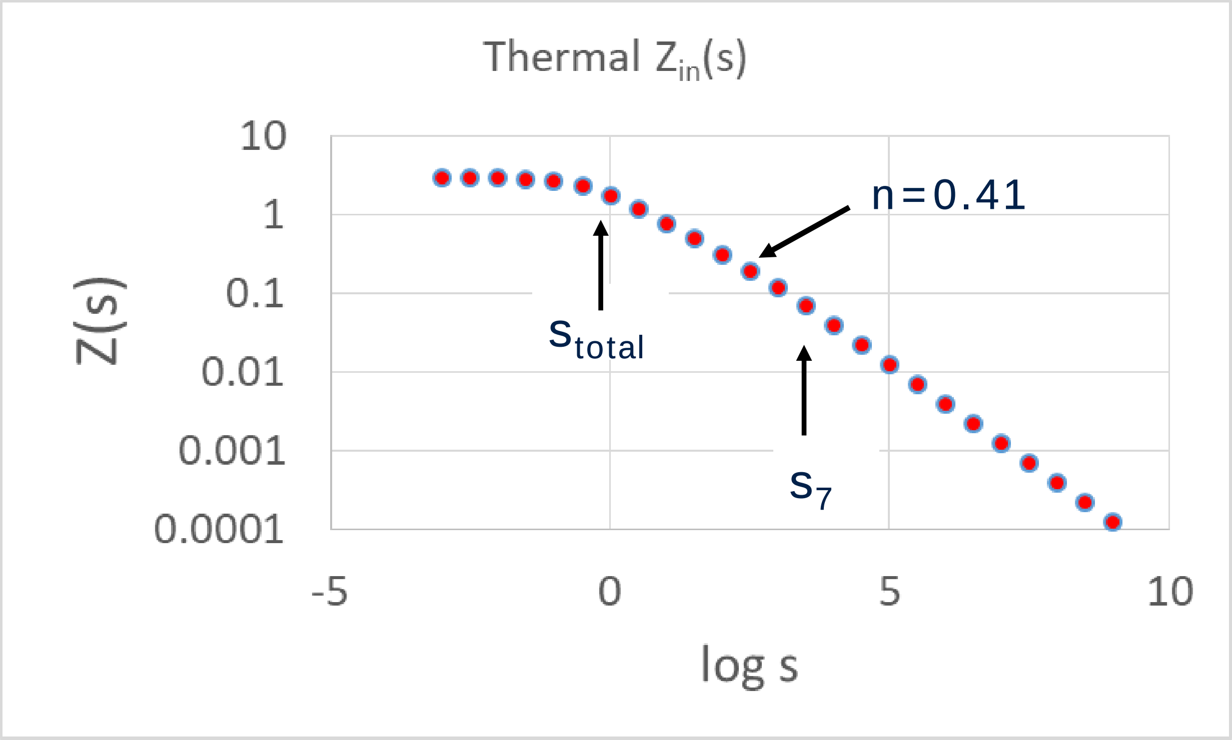
Figure 7: Thermal RC network, which is made matching the Wunsch-Bell curves.

Full Z(s) plot shows thermal τ effect

rx=0.673 (16x reduction in 7 stages)

cx=0.453 (256x reduction in 7 stages)

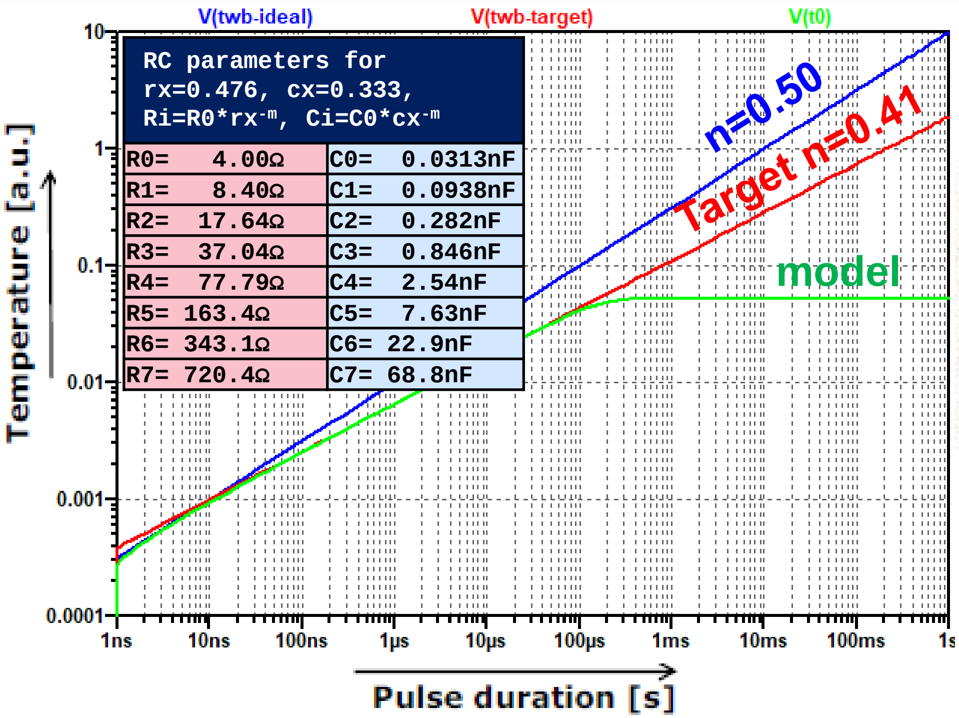
Heat sink effect seen below s_{total}

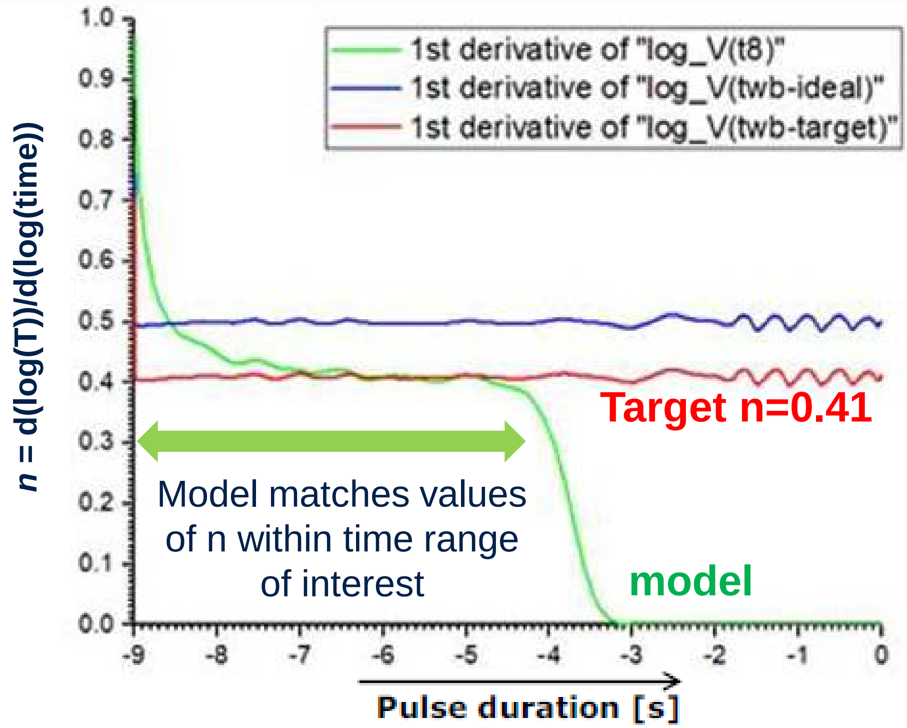


$\tau_i = R_i C_i \quad s_i = 1/\tau_i \quad \tau_{total} = \Sigma\tau_i \quad s_{total} = 1/\tau_{total}$

- Impedance taper ($rx/cx \neq 1$) causes departure of power curve fit from $n=-0.5$ (to $n=-0.41$), as found by Russ18 (3A.3), in s-range bracketing the thermal delay times of the network.

Test of lumped RC model parameters rx and cx





- Parameter values re-calculated over [Russ18] for target slope of n=0.41. Using 8-stage model, rx=0.476, cx=0.333, $y_r=\sqrt{(cx/rx)}$, reflection coefficient $\rho=(y_r-1)/(y_r+1)$, $n=0.5+\rho$ yields a calculated value of n=0.41092 which proves the validity of the TL-model.
- Simulations with R and C values demonstrate a good match with n=0.41 for t=100us…10ns. For t=10ns…1ns, n increases up to 1 which indicates the transition to the adiabatic regime.
- Some numerical noise originates from the accuracy of the general purpose circuit simulator and from determining the derivatives from the simulated temperatures. This is even most pronounced for the auxiliary values for n=0.41 and 0.5, which are both generated from exponential functions of the type $T=c*t^{-n}$.

Metal and heat sinks

Treated in 2016 (see esd16 reference) and earlier publications

$$Z(s) = \frac{1}{s + a\sqrt{s}\coth(k\sqrt{s})}. \quad (16)$$

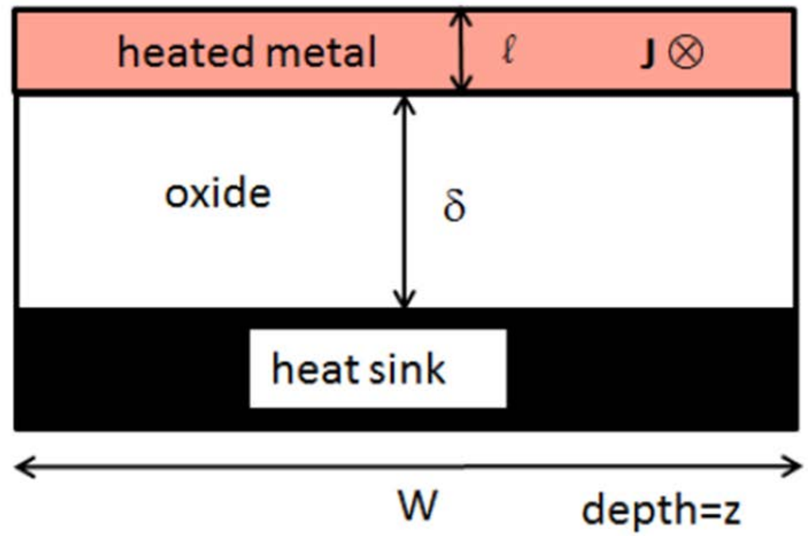


Figure 9. Metal on oxide with a heat sink, heated by electric current. Large values of W and z allow the 1-D approximation.

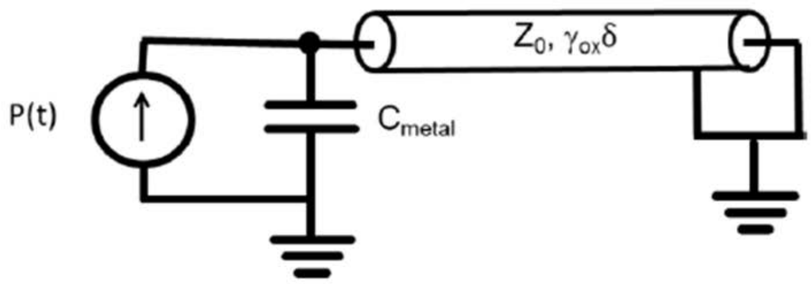


Figure 10. Electrical equivalent of metal on oxide with a heat sink, showing capacitive influence of bulk metal, and with I^2R power input as current source.

Surface capacitance and heat sinks

It is common to have adiabatic material (e.g., metal) at the surface of the device. This behaves like capacitance, i.e., Z~1/sC

$$Y_{in}(s) = C_0 s + \sqrt{\frac{Cs}{R}}$$

infinite slab

$$Y_{in}(s) = C_0 s + \sqrt{\frac{Cs}{R}}\coth(\sqrt{RCs} \cdot L)$$

shorted heat sink at depth L

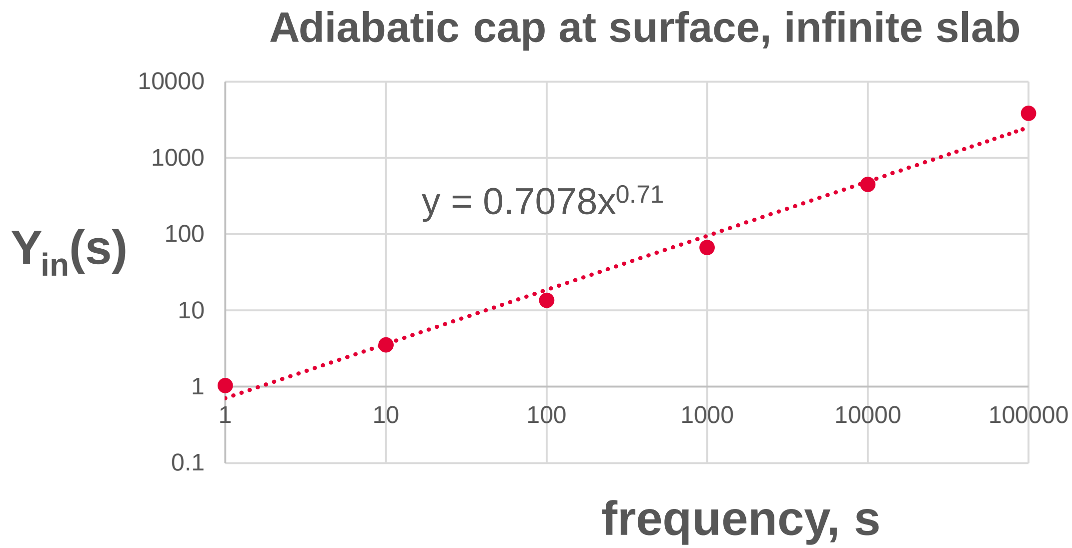
$Y_{in}=1/Z_{in}$, n now positive

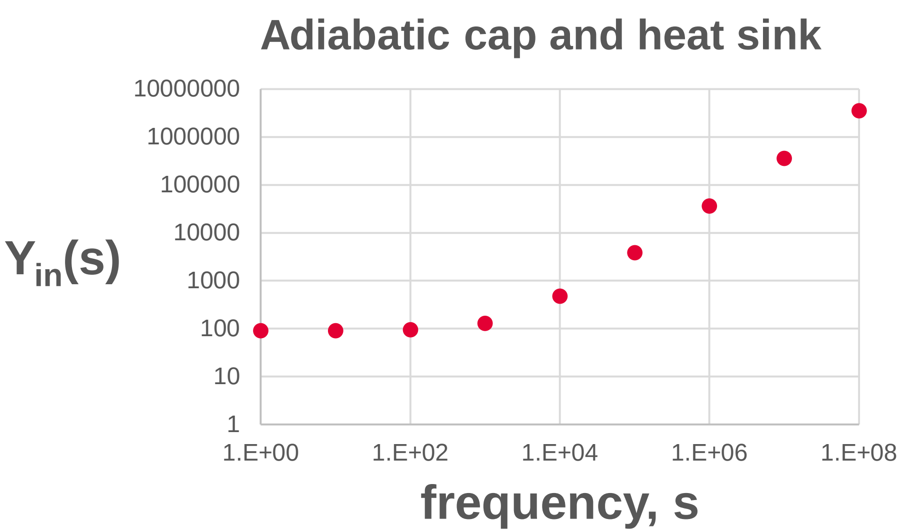
For the infinite slab, we expect $0.5<n<1$ for "moderate" values of frequency s

For the shorted heat sink, we expect $n=0$ at low frequency and a gradual transition to $n=1$ at very high s

Effect of adiabatic cap, optional heat sink

n=0.71 exponent from Russ18 can be matched at "moderate" frequency with adiabatic cap at surface





Heat sink causes Y_{in} to converge to a constant at low s, as expected

High s will still cause n→1

Continuously Tapered T-lines

Ragan, p. 305

6·1. Tapers in Coaxial Lines.—One of the simplest ways of joining two dissimilar coaxial lines is by means of an intermediate taper section which introduces the change in line characteristics gradually. A simple example of such a taper is given in Fig. 6·1. If the change occurs gradually enough only negligible reflected waves should be generated. To a first approximation this expectation is realized, and indeed the reflected wave approaches zero as the taper length approaches infinity. Frank[1] has shown that the reflection coefficient for a taper of length d within which the line constants are slowly varying is approximately

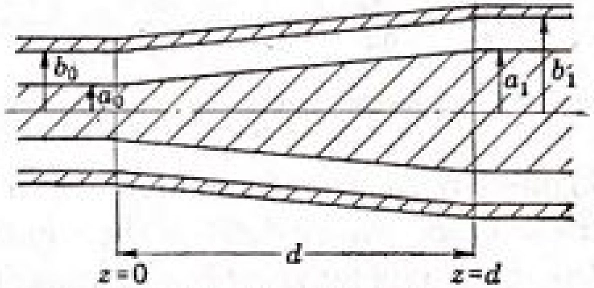


FIG. 6·1.—Coaxial line taper.

$$\Gamma = \frac{1}{4\gamma_0}\left[\frac{d(\ln Z)}{dz}\right]_0 - \frac{1}{4\gamma_d}\left[\frac{d(\ln Z)}{dz}\right]_d e^{-2\int_0^d \gamma\, dz}, \qquad (1)$$

where γ is the propagation constant, Z is the characteristic impedance, and the subscripts 0 and d denote values at the points $z = 0$ and $z = d$.

[1] N. H. Frank, "Reflections from Sections of Tapered Transmission Lines and Wave Guides," RL Report No. 189, Jan. 6, 1943.

305

Tapered RC line Input Impedance

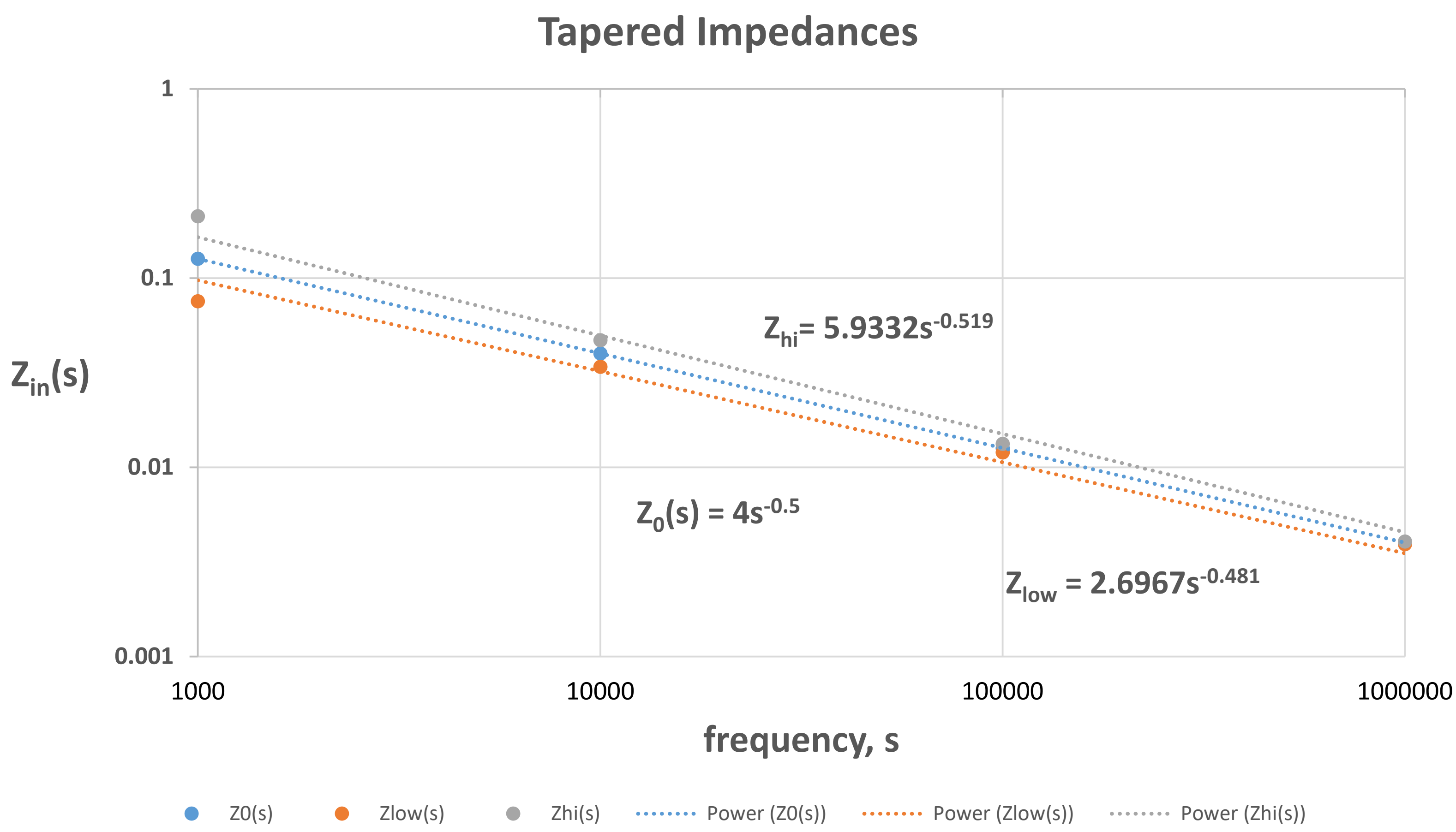
- Using the Frank formula, for a gently tapered RC line with substantial electrical length ($\Sigma RC > 1/s$), it can be shown that if

$$Z(s,x) = \sqrt{\frac{R}{Cs}}\, e^{\pm ax},$$

$$Z_{in}(s,0) \approx \sqrt{\frac{R}{Cs}\left[\frac{1 \pm \frac{a}{4\sqrt{RCs}}}{1 \mp \frac{a}{4\sqrt{RCs}}}\right]}$$

- The apparent semilog slope is as expected; $0<n<0.5$ for decreasing Z(x), and $0.5<n<1$ for increasing Z(x)
  - However, for small $a/\sqrt{RCs}$, a series in powers of $\sqrt{s}$ results and could be fit to find taper constant a.

$Z_{in}(s)$ for mildly tapered lines



Tapered line power laws for a=±0.5 for Zhi and Zlow

Effect is as expected, but mild because there is no shorting heat sink

Improving the continuous taper model

- We would not need the segmented, recursive model if we could treat continuous tapers of all kinds, and with terminating heat sinks, where Z=0 or a finite low value.
  - But the t-line math becomes a non-linear Riccati equation, with no analytic solution:

$$\frac{d\Gamma}{dx} - 2\gamma\Gamma + \frac{1}{2}\left[\frac{d(\ln Z_0)}{dx}\right](1-\Gamma^2) = 0$$

  - Numerical solution should be feasible in, e.g., Mathematica
  - However, a short can be analytically modeled for s high enough to allow a small but meaningful correction (next slide)

Tapered RC line with short

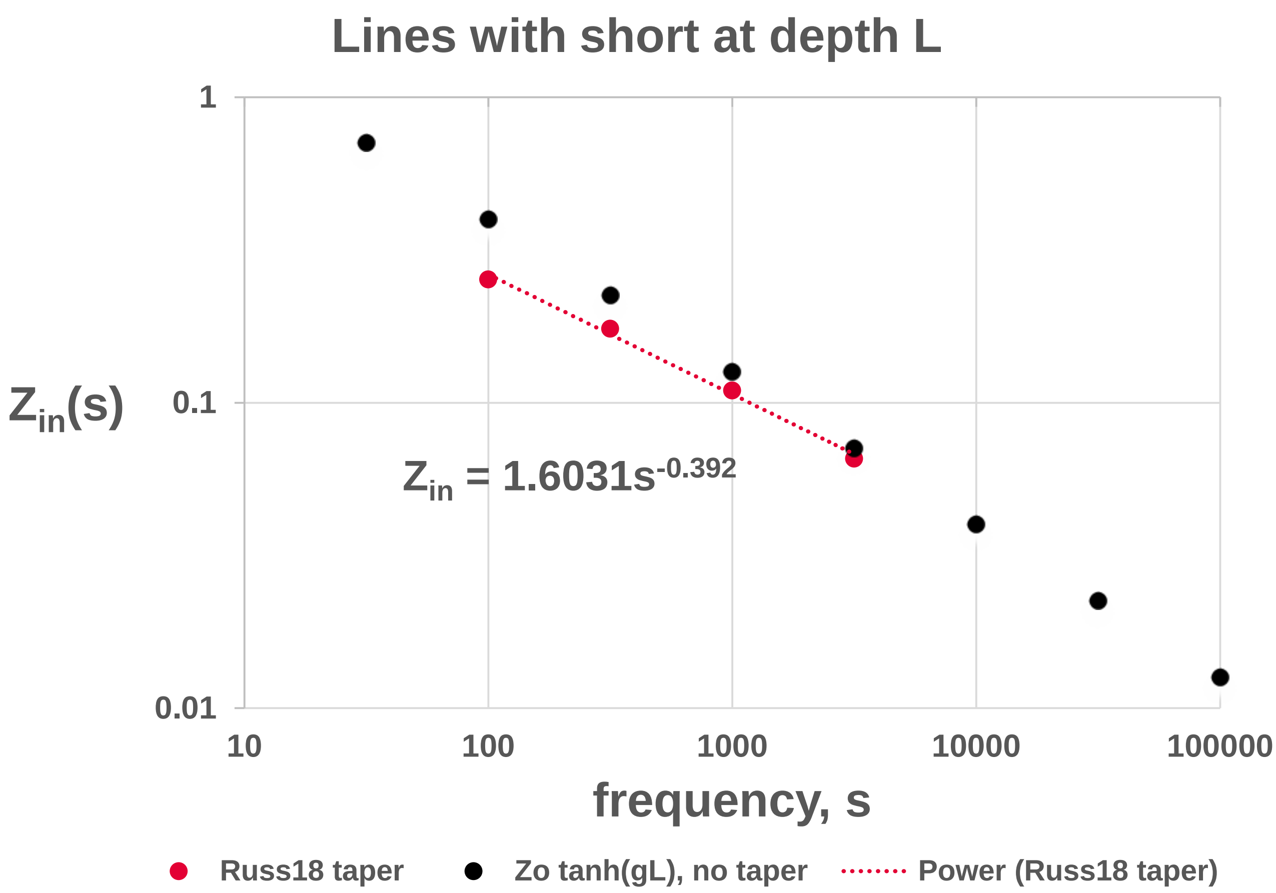
- For the range of s allowing a small correction, it can be shown that for a tapered line with a short at depth L,

$$Z_{in}(s,0) \approx \sqrt{\frac{R}{Cs}}\left[\frac{\tanh(\gamma L) \pm \frac{a}{4\sqrt{RCs}}}{1 \mp \frac{a}{4\sqrt{RCs}}}\right]$$

where $\gamma L = \sqrt{s}\sum_{i}\sqrt{R_i C_i}$

- For the Russ18 taper corresponding to n=0.41 exponent, calculations agree with the 8-segment recursive model, earlier slides (see power law fit on next slide)
  - Again, a series expansion of the above in √s terms may fit data even better

$Z_{in}(s)$ for shorted lines



- n=0.392 and Z_{in} values are close to 8-stage model, Russ18 lumped model, and observed data for tapered Z
- Convergence to tanh curve for short with no taper and n=0.5 is shown for comparison

Conclusions

- Like Russ18, we find for rx/cx>1, n<0.5; for rx/cx<1, n>0.5. This is believed due to the Z-mismatch between segments in this 8-stage model.
  - Trend is also confirmed with continuous taper theory
    - At high enough frequency, n→0.5 even for a tapered line
  - Reflection coefficient $\rho=(y_r-1)/(y_r+1)$; $y_r=\sqrt{(cx/rx)}$, leads to data fit (and conjecture) of $n \approx 0.5+\rho$ for Russ18 cases of n=0.41 and n=0.7. But 8 segments may be needed for good fit.
  - Cases of 0.5<n<1 also explained by surface adiabatic material
- Thermal time constant range of segmented tapered line determines critical range of s for power law
  - Elmore Delay concepts (see Gupta reference) give guidance
    - $\Sigma R_i C_i$ is most important for determining this range for s

Conclusions (cont'd)

- Adiabatic material (e.g., metal) at surface adds a low Z at the input; $|n|=1$ behavior at high s, short time pulses
  - Can explain equivalent $0.5<n<1$ since Z goes up subsurface
- Full solution to continuous taper and load (heat sink) could be enlightening—but must solve a Riccati equation

Important Takeaways:

- If Z(s) goes as $1/s^n$, the related Dwyer power curve goes as $1/t^n$.
- For $s>1/RC$ of the first segment, Z(s) reverts to $1/s^{0.5}$ because the time is too short to feel the influence of deeper Z-mismatched segments. Same is even true of a continuous taper at high s.
  - Similarly, steady state is reached at s below the (inverse) time constant of the entire structure.
- With these guides, thermal RC networks can be more readily synthesized given a particular lab result.

References

- T.J. Maloney, "Unified Model of 1-D Pulsed Heating, Combining Wunsch-Bell with the Dwyer Curve", 2016 EOS/ESD Symposium, paper 7A.2. See https://sites.google.com/site/esdpubs/documents/esd16.pdf.
- R. Gupta, et al., "The Elmore Delay as a Bound for RC Trees with Generalized Input Signals", IEEE Trans. On Computer-Aided Design of Integrated Circuits and Systems, Jan. 1997, pp. 95-104.
- G.L. Ragan, *Microwave Transmission Circuits*, 1964, pp. 305-311. Volume 9 of MIT Radiation Lab series, available in .pdf at http://www.jlab.org/ir/MITSeries.html.